



# Intermediate Algebra

ELEVENTH EDITION

LIAL | HORNSBY | MCGINNIS

Eleventh  
Edition

# Intermediate Algebra

**Margaret L. Lial**

American River College

**John Hornsby**

University of New Orleans

**Terry McGinnis**



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Manufacturing Buyer:	Carol Melville, LSC Communications
Associate Director of Design:	Blair Brown
Program Design Lead:	Barbara Atkinson
Text Design, Production Coordination, Composition, and Illustrations:	Cenveo® Publisher Services
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To Dotty, Puddles, and Gus. You loved us unconditionally.

E.J.H. and T.R.M.

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# Preface

It is with great pleasure that we offer the eleventh edition of *Intermediate Algebra*. We have remained true to the original goal that has guided us over the years—to provide the best possible text and supplements package to help students succeed and instructors teach. This edition faithfully continues that process through enhanced explanations of concepts, new and updated examples and exercises, student-oriented features like Pointers, Cautions, Problem-Solving Hints, Margin Problems, and Study Skills, as well as an extensive package of helpful supplements and study aids.

This text is part of a series that also includes the following books:

- *Basic College Mathematics*, Tenth Edition, by Lial, Salzman, and Hestwood
- *Prealgebra*, Sixth Edition, by Lial and Hestwood
- *Introductory Algebra*, Eleventh Edition, by Lial, Hornsby, and McGinnis
- *Introductory and Intermediate Algebra*, Sixth Edition, by Lial, Hornsby, and McGinnis
- *Developmental Mathematics: Basic Mathematics and Algebra*, Fourth Edition, by Lial, Hornsby, McGinnis, Salzman, and Hestwood

## WHAT'S NEW IN THIS EDITION

We are pleased to offer the following new text features and supplements.

- ▶ **Revised Exposition** With each edition of the text, we continue to polish and improve discussions and presentations of topics to increase readability and student understanding. We believe this edition is the best yet in this regard.
- ▶ **More Figures and Diagrams** For visual learners, we have made a concerted effort to add mathematical figures, diagrams, tables, and graphs whenever possible.
- ▶ **Enhanced Use of Pedagogical Color** We have thoroughly reviewed all pedagogical color in discussions and examples and increased its use wherever doing so would enhance concept development, emphasize important steps, or highlight key information.
- ▶ **Improved Study Skills** Most of these special activities now include a **Now Try This** section to increase student involvement. Each is designed independently to allow flexible use with individuals or small groups of students, or as a source of material for in-class discussions.
- ▶ **More What Went Wrong? Exercises** We have increased the number of these popular **CONCEPT CHECK** exercises, which highlight common student errors.
- ▶ **More Relating Concepts Exercises** We have increased the number of these flexible groups of exercises, located at the end of many exercise sets. Specially written to help students tie concepts together, as well as compare and contrast ideas, identify and describe patterns, and extend concepts to new situations, these sets of problems may be used with individual students or collaboratively with pairs or small groups. All of these exercise sets have been added to MyMathLab and tagged for easy location and assignment.
- ▶ **Dedicated Mixed Review Exercises** Each chapter review has been expanded to include a one-page set of Mixed Review Exercises to help students further synthesize concepts.
- ▶ **Learning Catalytics** This interactive student response tool uses students' own devices to engage them in the learning process. Learning Catalytics is accessible through MyMathLab and can be customized to an instructor's specific needs. Instructors can employ this tool to generate class discussion, promote peer-to-peer learning, and use real-time data to adjust instructional strategy. As an introduction to this exciting new tool, we have provided prerequisite skills questions at the beginning of each section to check students' preparedness for the new section. Learn more about Learning Catalytics in the Instructor Resources tab in MyMathLab.



- ▶ **Enhanced MyMathLab Resources** Exercise coverage has been refined with new videos and homework problems, including new Relating Concepts questions added throughout the course. See pages x and xi for more details.
- ▶ **Data Analytics** We analyzed aggregated student usage and performance data from MyMathLab for the previous edition of this text. The results of this analysis helped us improve the quality and quantity of exercises that matter the most to instructors and students.

## CONTENT CHANGES


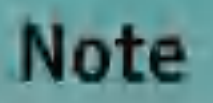




Specific content changes include the following:

- ▶ **Exercise sets** have been updated with a renewed focus on conceptual understanding, skill development, and review. New or revised figures are included wherever possible.
- ▶ **Real-world data** in the examples and exercises has been updated.
- ▶ **Sections 1.4, 2.3, and 4.3** include new problem-solving objectives, examples, exercises, and/or hints. **Section 1.2** provides new formulas from the health care industry.
- ▶ **The presentation on linear equations in two variables in Section 3.1** has been reorganized. New exercises have been added in Sections 3.1–3.3 to make better connections among tables, equations, and graphs of linear equations.
- ▶ **The introduction to relations and functions in Section 3.5** has a new example and expanded discussion on relations and ordered pairs.
- ▶ **Scientific notation is covered separately in Section 5.2.**
- ▶ **Increased connections between operations with rational numbers and operations with rational expressions** are made in Sections 7.1–7.3.
- ▶ **The following topics are among those that have been enhanced and/or expanded:**
  - Solving linear equations in one variable with fraction and decimal coefficients (Section 1.1)
  - Graphing linear inequalities in two variables (Section 3.4)
  - Solving systems of linear equations in three variables (Section 4.2)
  - Factoring sums and differences of cubes (Section 6.3)
  - General factoring strategies (Section 6.4)
  - Determining domains of rational functions (Sections 7.1, 7.4)
  - Solving rational equations (Section 7.4)
  - Multiplying radical expressions (Section 8.5)
  - Solving quadratic equations using the zero-factor property and the quadratic formula (Sections 9.1, 9.2)
  - Solving quadratic inequalities (Section 9.7)
  - Finding and graphing inverse functions (Section 10.2)
  - Graphing systems of linear inequalities (Section 11.5)

## HALLMARK FEATURES

We have enhanced the following popular features, each of which is designed to increase ease of use by students and/or instructors.

- ▶ **Emphasis on Problem-Solving** We introduce our six-step problem-solving method in Chapter 1 and integrate it throughout the text. The six steps, *Read*, *Assign a Variable*, *Write an Equation*, *Solve*, *State the Answer*, and *Check*, are emphasized in boldface type and repeated in examples and exercises to reinforce the problem-solving process for students. We also provide students with **Problem-Solving Hint** boxes that feature helpful problem-solving tips and strategies.

- ▶ **Helpful Learning Objectives** We begin each section with clearly stated, numbered objectives, and the included material is directly keyed to these objectives so that students and instructors know exactly what is covered in each section.
- ▶ **Popular Cautions and Notes** One of the most popular features of previous editions, we include information marked  **CAUTION** and  **Note** to warn students about common errors and emphasize important ideas throughout the exposition. The updated text design makes them easy to spot.
- ▶ **Comprehensive Examples** The new edition features a multitude of step-by-step, worked-out examples that include pedagogical color, helpful side comments, and special pointers. We give special attention to checking example solutions—more checks, designated using a special **CHECK** tag and ✓, are included than in past editions.
- ▶ **Guided Solutions** Selected exercises in the margins and in the exercise sets, marked with a  icon, show the first few solution steps. Many of these exercises can be found in the MyMathLab homework, providing guidance to students as they start learning a new concept or procedure.
- ▶ **More Pointers** Because they were so well received by both students and instructors in the previous edition, we incorporate more pointers in examples and discussions throughout this edition of the text. They provide students with important on-the-spot reminders and warnings about common pitfalls.
- ▶ **Ample Margin Problems** Margin problems, with answers immediately available at the bottom of the page, are found in every section of the text. This key feature allows students to immediately practice the material covered in the examples in preparation for the exercise sets. Many include guided solutions.
- ▶ **Updated Figures, Photos, and Hand-Drawn Graphs** Today's students are more visually oriented than ever. As a result, we include appealing mathematical figures, diagrams, tables, and graphs, including a “hand-drawn” style of graphs, whenever possible. We have incorporated depictions of well-known mathematicians as well as photos to accompany applications in examples and exercises.
- ▶ **Relevant Real-Life Applications** We include many new or updated applications from fields such as business, pop culture, sports, technology, and the health sciences that show the relevance of algebra to daily life.
- ▶ **Extensive and Varied Exercise Sets** The text contains a wealth of exercises to provide students with opportunities to practice, apply, connect, review, and extend the skills they are learning. Numerous illustrations, tables, graphs, and photos help students visualize the problems they are solving. Problem types include skill-building and writing exercises, as well as applications, matching, true/false, multiple-choice, and fill-in-the-blank problems.  
  
In the Annotated Instructor's Edition of the text, the writing exercises are marked with an icon  so that instructors may assign these problems at their discretion. Students can watch an instructor work through the complete solution for all exercises marked with a Play Button icon  in MyMathLab.
- ▶ **Special Summary Exercises** We include a set of these popular in-chapter exercises in many chapters. They provide students with the all-important *mixed review problems* they need to master topics and often include summaries of solution methods and/or additional examples.
- ▶ **Step-by-Step Solutions to Selected Exercises** Exercise numbers enclosed in a blue square, such as  **11.**, indicate that a worked-out solution for the problem is available in MyMathLab. These solutions are given for selected exercises that most commonly cause students difficulty.

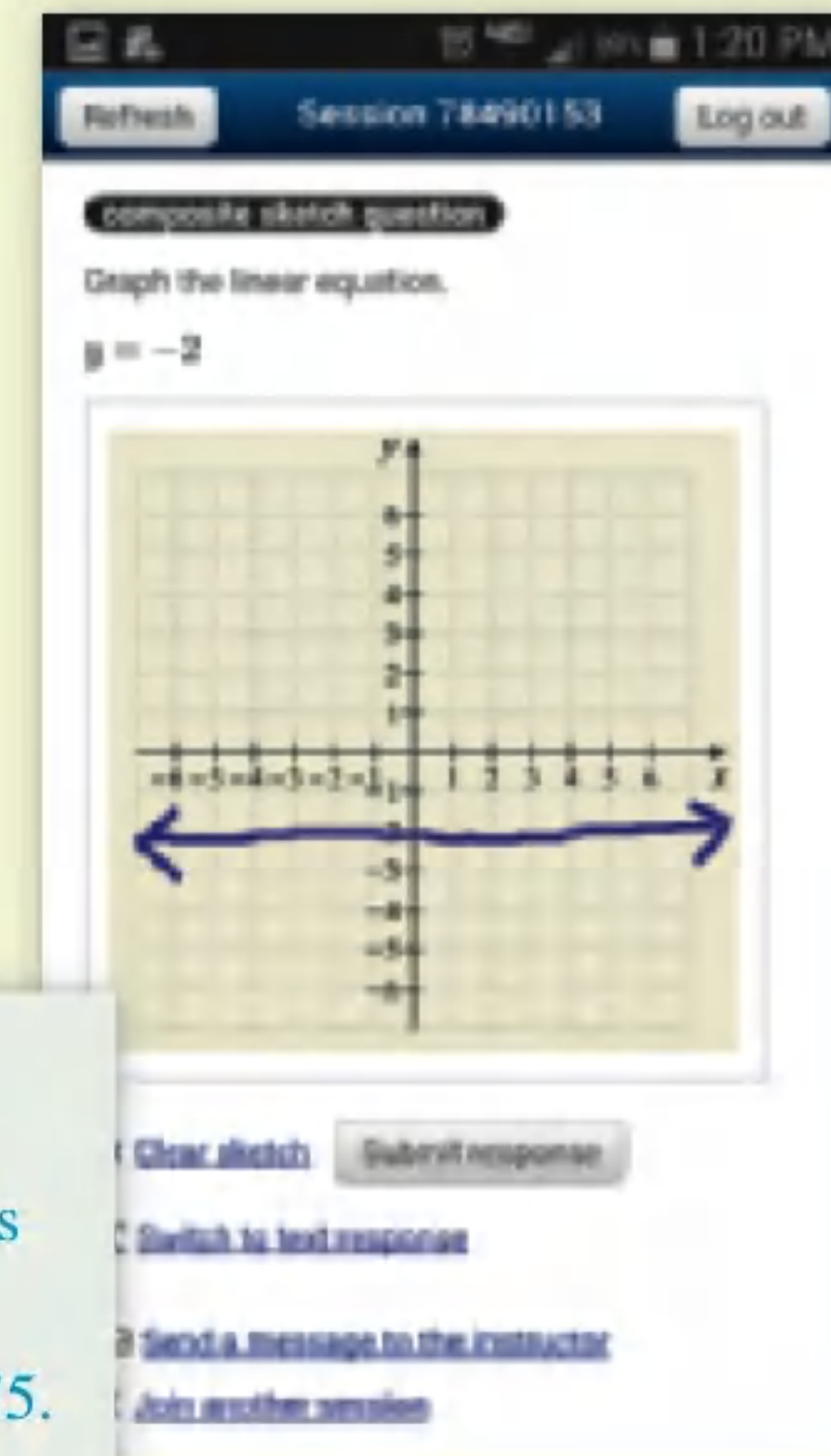
# Resources for Success

**MyMathLab Online Course** for Lial/Hornsby/  
McGinnis *Intermediate Algebra*, 11th edition

The corresponding MyMathLab course tightly integrates the authors' approach, giving students a learning environment that encourages conceptual understanding and engagement.

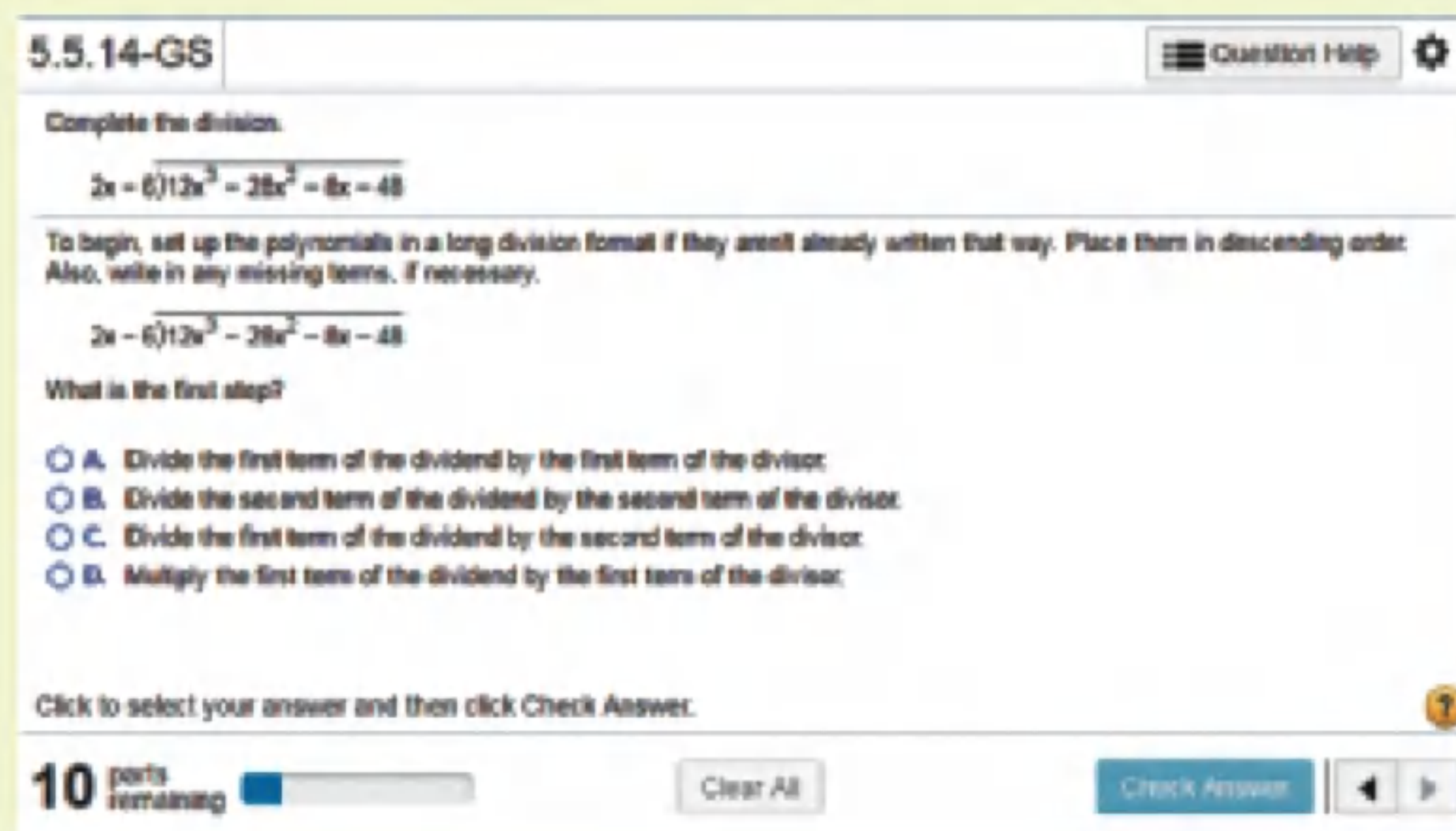
## NEW! Learning Catalytics

Integrated into MyMathLab, Learning Catalytics use students' mobile devices for an engagement, assessment, and classroom intelligence system that gives instructors real-time feedback on student learning. LC annotations for instructors in the text provide corresponding questions that they can use to engage their classrooms.



### LC LEARNING CATALYTICS

1. Which digit in the number 40,163 is in the ones place?
2. Identify the place value of 8 in the number 9875.



## NEW! Workspace Assignments

These new assignments allow students to naturally write out their work by hand, step-by-step, showing their mathematical reasoning as they receive instant feedback at each step. Each student's work is captured in the MyMathLab gradebook so instructors can easily pinpoint exactly where in the solution process students struggled.

## Expanded! Conceptual Exercises

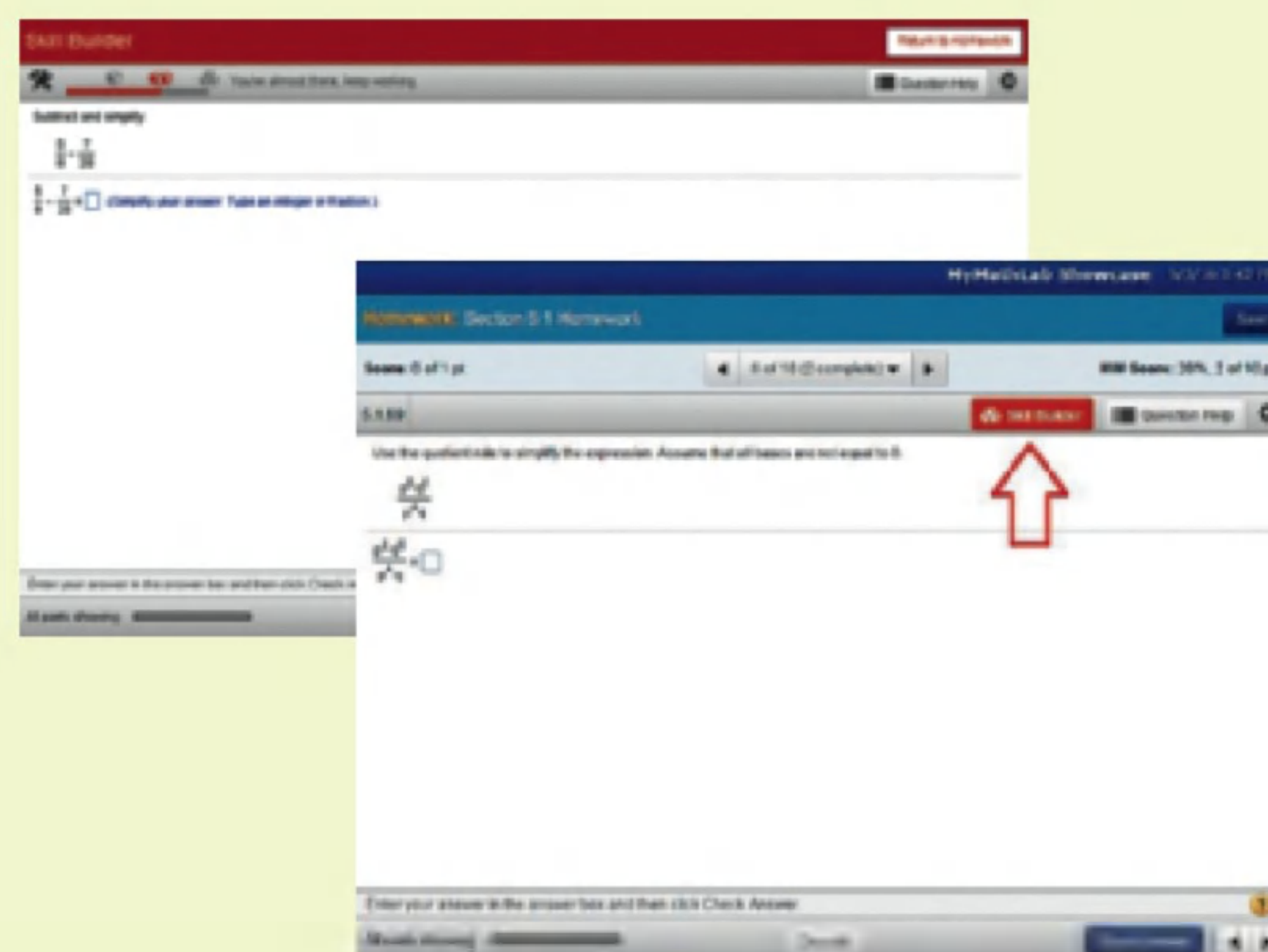
In addition to MyMathLab's hallmark interactive exercises, the Lial team provides students with exercises that tie concepts together and help students problem-solve. Guided Solutions exercises, marked with a "GS" in the Assignment Manager, test student understanding of the problem-solving steps while guiding them through the solution process. Relating Concepts exercises in the text help students make connections and problem-solve at a higher level. These sets are assignable in MyMathLab, with expanded coverage.



# Resources for Success

## NEW! Adaptive Skill Builder

When students struggle on an exercise, Skill Builder assignments provide just-in-time, targeted support to help them build on the requisite skills needed to complete their assignment. As students progress, the Skill Builder assignments adapt to provide support exercises that are personalized to each student's activity and performance throughout the course.



## Instructor Resources

### Annotated Instructor's Edition

**ISBN 10:** 0-13-445616-5 **ISBN 13:** 978-0-13-445616-4  
The AIE provides annotations for instructors, including answers, Learning Catalytics suggestions, and vocabulary and teaching tips.

The following resources can be downloaded from [www.pearsonhighered.com](http://www.pearsonhighered.com) or in MyMathLab:

### Instructor's Solutions Manual

This manual provides solutions to all exercises in the text.

### Instructor's Resource Manual

This manual includes Mini-Lectures to provide new instructors with objectives, key examples, and teaching tips for every section of the text.

### PowerPoints

These slides, which can be edited, present key concepts and definitions from the text.

### TestGen

TestGen® ([www.pearsoned.com/testgen](http://www.pearsoned.com/testgen)) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

## Student Resources

### Student Solutions Manual

**ISBN 10:** 0-13-444595-3 **ISBN 13:** 978-0-13-444595-3  
This manual contains completely worked-out solutions for all the odd-numbered exercises in the text.

### Lial Video Workbook

**ISBN 10:** 0-13-445613-0 **ISBN 13:** 978-0-13-445613-3  
This workbook/note-taking guide helps students develop organized notes as they work along with the videos. The notebook includes

- Guided Examples to be used in conjunction with the Lial Section Lecture Videos and/or Objective-Level Video clips, plus corresponding Now Try This exercises for each text objective.
- Extra practice exercises for every section of the text, with ample space for students to show their work.
- Learning objectives and key vocabulary terms for every text section, along with vocabulary practice problems.

## ACKNOWLEDGMENTS

The comments, criticisms, and suggestions of users, nonusers, instructors, and students have positively shaped this text over the years, and we are most grateful for the many responses we have received. The feedback gathered for this revision of the text was particularly helpful, and we especially wish to thank the following individuals who provided invaluable suggestions for this and the previous editions:

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We particularly thank the many students and instructors who have used this text over the years. You are the reason we do what we do. It is our hope that we have positively impacted your mathematics journey. We would welcome any comments or suggestions you might have via email to [math@pearson.com](mailto:math@pearson.com).

John Hornsby  
 Terry McGinnis

# R

# Review of the Real Number System

**R.1** Basic Concepts

**R.2** Operations on Real Numbers

**R.3** Exponents, Roots, and Order of Operations

**R.4** Properties of Real Numbers

**Study Skills** *Using Your Math Text*

**Study Skills** *Reading Your Math Text*

## R.1 Basic Concepts

**OBJECTIVE** 1 Write sets using set notation. A set is a collection of objects called the **elements**, or **members**, of the set. In algebra, the elements of a set are usually numbers. Set braces,  $\{ \}$ , are used to enclose the elements.

For example, 2 is an element of the set  $\{1, 2, 3\}$ . Because we can count the number of elements in the set  $\{1, 2, 3\}$ , it is a **finite set**.

In algebra, we refer to certain sets of numbers by name. The set

$$N = \{1, 2, 3, 4, 5, 6, \dots\} \quad \text{Natural (counting) numbers}$$

is the **natural numbers**, or the **counting numbers**. The three dots (*ellipsis points*) show that the list continues in the same pattern indefinitely. We cannot list all of the elements of the set of natural numbers, so it is an **infinite set**.

Including 0 with the set of natural numbers gives the set of **whole numbers**.

$$W = \{0, 1, 2, 3, 4, 5, 6, \dots\} \quad \text{Whole numbers}$$

The set containing no elements is the **empty set**, or **null set**, usually written  $\emptyset$ . For example, the set of whole numbers less than 0 is  $\emptyset$ .

### CAUTION

Do not write  $\{\emptyset\}$  for the empty set.  $\{\emptyset\}$  is a set with one element,  $\emptyset$ . Use the notation  $\emptyset$  for the empty set.

Work Problem 1 at the Side. ►

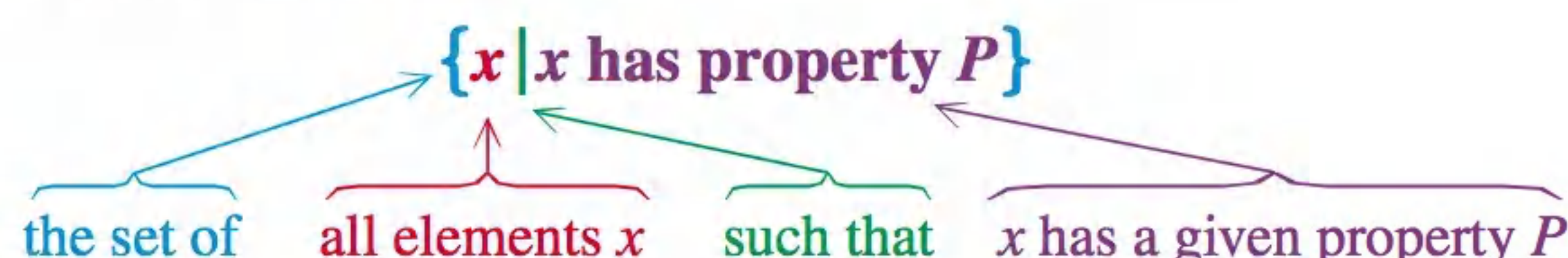
A **variable** is a symbol, usually a letter, used to represent an unknown number or to define a set of numbers. For example,

$$\{x \mid x \text{ is a natural number between 3 and 15}\}$$

(read “the set of all elements  $x$  such that  $x$  is a natural number between 3 and 15”) defines the following set.

$$\{4, 5, 6, 7, \dots, 14\}$$

The notation  $\{x \mid x \text{ is a natural number between 3 and 15}\}$  is an example of **set-builder notation**.



## OBJECTIVES

- 1 Write sets using set notation.
- 2 Use number lines.
- 3 Know the common sets of numbers.
- 4 Find additive inverses.
- 5 Use absolute value.
- 6 Use inequality symbols.

- 1 Consider the set.

$$\left\{0, 10, \frac{3}{10}, 52, 98.6\right\}$$

- (a) Which elements of the set are natural numbers?

- (b) Which elements of the set are whole numbers?

### Answers

1. (a) 10 and 52 (b) 0, 10, and 52

2 List the elements in each set.

(a)  $\{x \mid x \text{ is a whole number less than } 5\}$

(b)  $\{y \mid y \text{ is a natural number greater than } 12\}$

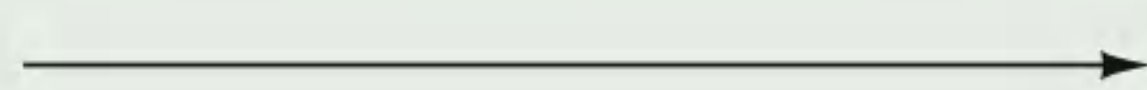
3 Use set-builder notation to describe each set.

(a)  $\{0, 1, 2, 3, 4, 5\}$

(b)  $\{7, 14, 21, 28, \dots\}$

4 Graph the elements of each set.

(a)  $\{-4, -2, 0, 2, 4, 6\}$



(b)  $\{-1, 0, \frac{2}{3}, 2.5\}$



(c)  $\{5, \frac{16}{3}, 6, \frac{13}{2}, 7, \frac{29}{4}\}$



**EXAMPLE 1** Listing the Elements in Sets

List the elements in each set.

(a)  $\{x \mid x \text{ is a natural number less than } 4\}$

The natural numbers less than 4 are 1, 2, and 3. This set is  $\{1, 2, 3\}$ .

(b)  $\{y \mid y \text{ is one of the first five even natural numbers}\}$  is  $\{2, 4, 6, 8, 10\}$ .

(c)  $\{z \mid z \text{ is a natural number greater than or equal to } 7\}$

The set of natural numbers greater than or equal to 7 is an infinite set, written with ellipsis points as

$$\{7, 8, 9, 10, \dots\}.$$

◀ Work Problem 2 at the Side.

**EXAMPLE 2** Using Set-Builder Notation to Describe Sets

Use set-builder notation to describe each set.

(a)  $\{1, 3, 5, 7, 9\}$

There are often several ways to describe a set with set-builder notation. One way to describe the given set is

$$\{x \mid x \text{ is one of the first five odd natural numbers}\}.$$

(b)  $\{5, 10, 15, \dots\}$

This set can be described as  $\{x \mid x \text{ is a positive integer multiple of } 5\}$ .

◀ Work Problem 3 at the Side.

**OBJECTIVE 2** Use number lines. A good way to get a picture of a set of numbers is to use a **number line**. See **Figure 1**.

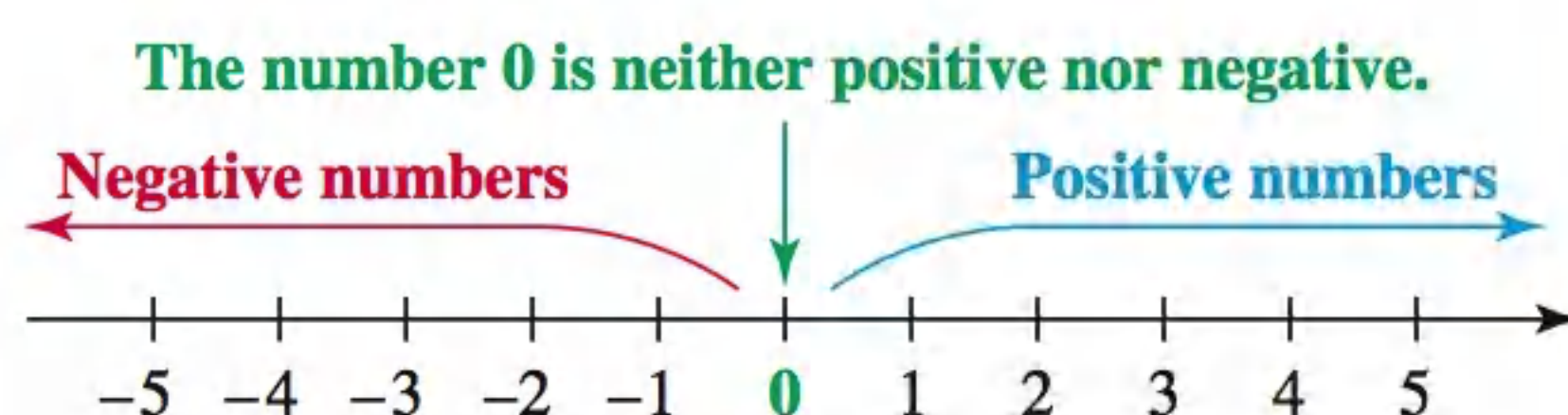


Figure 1

To draw a number line, choose any point on the line and label it 0. Then choose any point to the right of 0 and label it 1. Use the distance between 0 and 1 as the scale to locate, and then label, other points.

The set of numbers identified on the number line in **Figure 1**, including positive and negative numbers and 0, is part of the set of **integers**.

$$I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\} \quad \text{Integers}$$

Each number on a number line is the **coordinate** of the point that it labels, while the point is the **graph** of the number. **Figure 2** shows a number line with several selected points graphed on it.

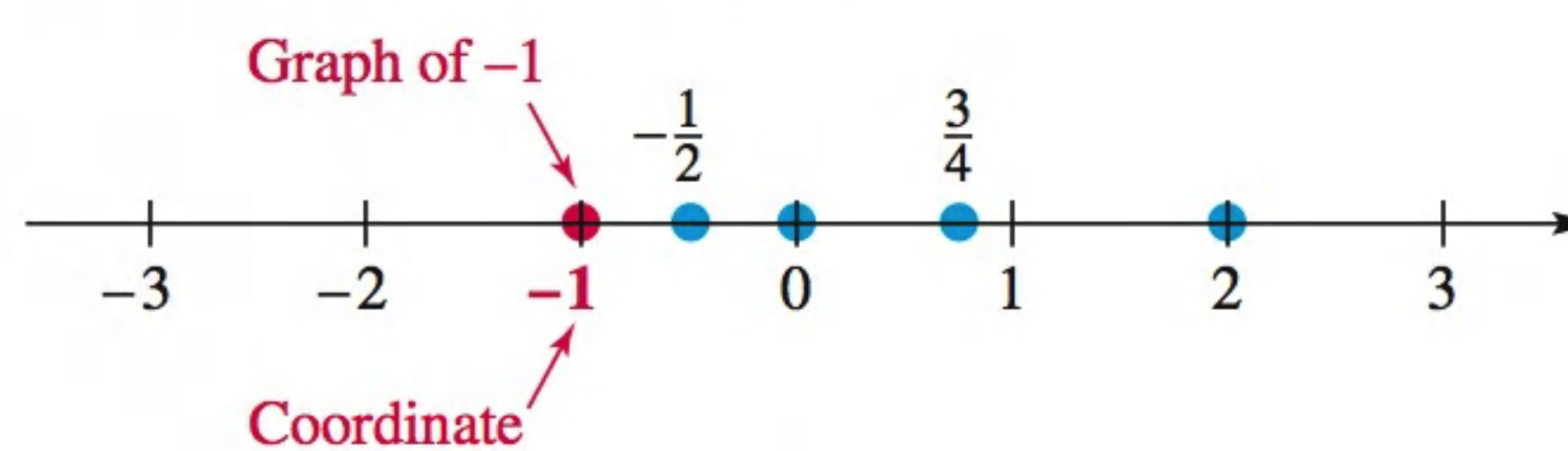


Figure 2

◀ Work Problem 4 at the Side.

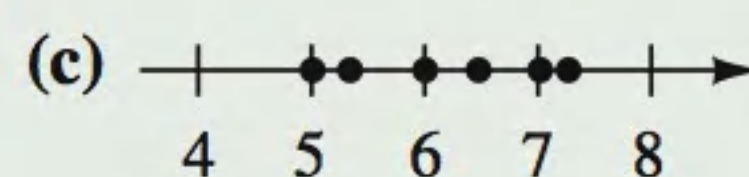
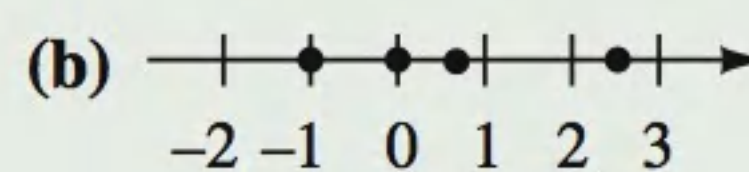
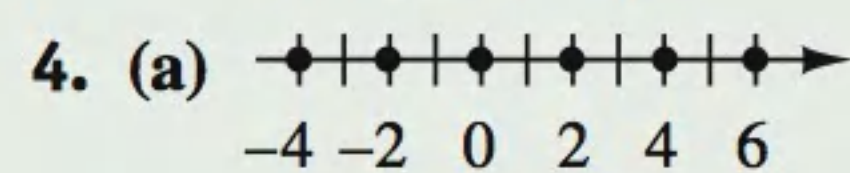
**Answers**

2. (a)  $\{0, 1, 2, 3, 4\}$

(b)  $\{13, 14, 15, \dots\}$

3. (a) One answer is  $\{x \mid x \text{ is a whole number less than } 6\}$ .

(b) One answer is  $\{x \mid x \text{ is a positive integer multiple of } 7\}$ .



The fractions  $-\frac{1}{2}$  and  $\frac{3}{4}$ , graphed on the number line in **Figure 2**, are examples of *rational numbers*. A **rational number** can be expressed as the quotient of two integers, with denominator not 0. The set of all rational numbers is written as follows.

$$\left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers, } q \neq 0 \right\} \quad \text{Rational numbers}$$

The set of rational numbers includes the natural numbers, whole numbers, and integers because these numbers can be written as fractions.

Examples:  $14 = \frac{14}{1}$ ,  $-3 = \frac{-3}{1}$ , and  $0 = \frac{0}{1}$

A rational number written as a fraction, such as  $\frac{1}{8}$  or  $\frac{2}{3}$ , can also be expressed as a decimal by dividing the numerator by the denominator.

$\begin{array}{r} 0.125 \leftarrow \text{Terminating decimal} \\ 8 \overline{)1.000} \quad \text{(rational number)} \\ \underline{8} \phantom{00} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \leftarrow \text{Remainder is 0.} \end{array}$ $\frac{1}{8} = 0.125$	$\begin{array}{r} 0.666\dots \leftarrow \text{Repeating decimal} \\ 3 \overline{)2.000\dots} \quad \text{(rational number)} \\ \underline{18} \phantom{00} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \leftarrow \text{Remainder is never 0.} \end{array}$ $\frac{2}{3} = 0.\overline{6} \leftarrow \text{A bar is written over the repeating digit(s).}$
---	---

Thus, terminating decimals, such as  $0.125 = \frac{1}{8}$ ,  $0.8 = \frac{4}{5}$ , and  $2.75 = \frac{11}{4}$ , and decimals that have a repeating block of digits, such as  $0.\overline{6} = \frac{2}{3}$  and  $0.\overline{27} = \frac{3}{11}$ , are rational numbers.

Decimal numbers that neither terminate nor repeat, which include many square roots, are *irrational numbers*.

$$\sqrt{2} = 1.414213562\dots \quad \text{and} \quad -\sqrt{7} = -2.6457513\dots \quad \text{Irrational numbers}$$

### Note

Some square roots, such as  $\sqrt{16} = 4$  and  $\sqrt{\frac{9}{25}} = \frac{3}{5}$ , are rational.

A decimal number such as  $0.010010001\dots$  has a pattern, but it is irrational because there is no fixed block of digits that repeats. Another irrational number is  $\pi$ . See **Figure 3**.

Some rational and irrational numbers are graphed on the number line in **Figure 4**. The rational numbers together with the irrational numbers make up the set of **real numbers**.

**Every point on a number line corresponds to a real number, and every real number corresponds to a point on the number line.**

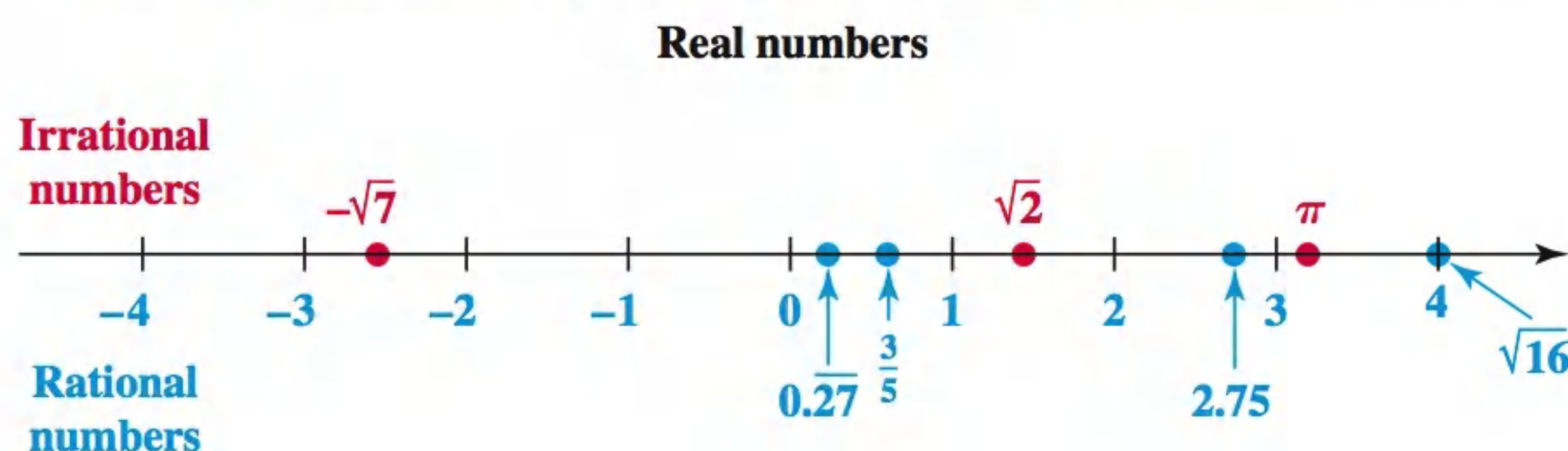
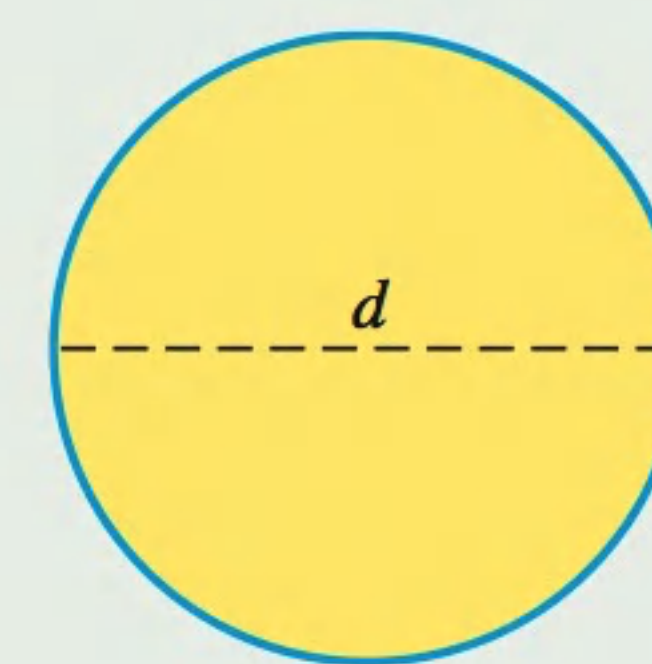


Figure 4



$$\pi = \frac{C}{d}$$

$\pi$ , the ratio of the circumference of a circle to its diameter, is approximately equal to 3.141592653....

Figure 3



**OBJECTIVE 3** Know the common sets of numbers.

Sets of Numbers	
Natural numbers	$\{1, 2, 3, 4, 5, 6, \dots\}$
Whole numbers	$\{0, 1, 2, 3, 4, 5, 6, \dots\}$
Integers	$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
Rational numbers	$\{\frac{p}{q}   p \text{ and } q \text{ are integers, where } q \neq 0\}$ Examples: $\frac{4}{1}, 1.3, -\frac{9}{2}, \frac{16}{8}$ or $2, \sqrt{9}$ or $3, 0.\bar{6}$
Irrational numbers	$\{x   x \text{ is a real number that cannot be represented by a terminating or repeating decimal}\}$ Examples: $\sqrt{3}, -\sqrt{2}, \pi, 0.010010001\dots$
Real numbers	$\{x   x \text{ is a rational or an irrational number}\}^*$

Figure 5 shows the set of real numbers. *Every real number is either rational or irrational.* Notice that the integers are elements of the set of rational numbers and that the whole numbers and natural numbers are elements of the set of integers.

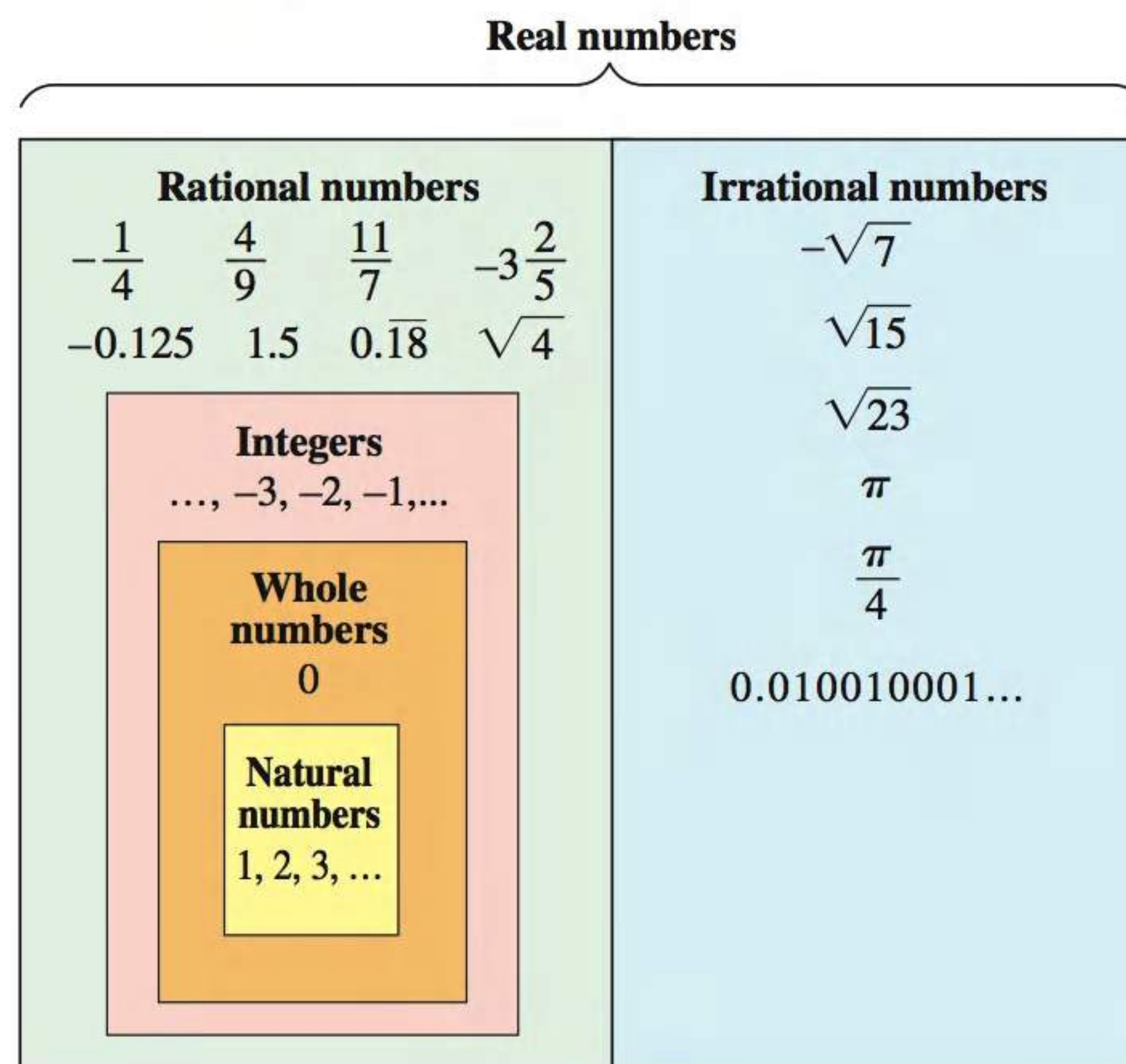


Figure 5

**EXAMPLE 3** Identifying Examples of Number Sets

List the numbers in the following set that are elements of each set.

$$\left\{-8, -\sqrt{2}, -\frac{9}{64}, 0, 0.5, \frac{2}{3}, 1.\bar{12}, \sqrt{3}, 2, \pi\right\}$$

- |   |  |
|---|--|
| (a) Integers<br>-8, 0, and 2                                | (b) Rational numbers<br>-8, $-\frac{9}{64}$ , 0, 0.5, $\frac{2}{3}$ , $1.\bar{12}$ , and 2 |
| (c) Irrational numbers<br>$-\sqrt{2}, \sqrt{3}$ , and $\pi$ | (d) Real numbers<br>All are real numbers.  |

◀ Work Problem 5 at the Side.

5 List the numbers in the following set that are elements of each set.

$$\left\{-2.4, -\sqrt{1}, -\frac{1}{2}, 0, 0.\bar{3}, \sqrt{5}, \pi, 5\right\}$$

(a) Whole numbers

(b) Rational numbers

(c) Irrational numbers

**Answers**

5. (a)  $\{0, 5\}$

(b)  $\left\{-2.4, -\sqrt{1}, -\frac{1}{2}, 0, 0.\bar{3}, 5\right\}$

(c)  $\{\sqrt{5}, \pi\}$

\*An example of a number that is not real is  $\sqrt{-1}$ . This number, part of the *complex number system*, is discussed later in the text.

**EXAMPLE 4** Determining Relationships between Sets of Numbers

Decide whether each statement is *true* or *false*. If it is false, tell why.

(a) All irrational numbers are real numbers.

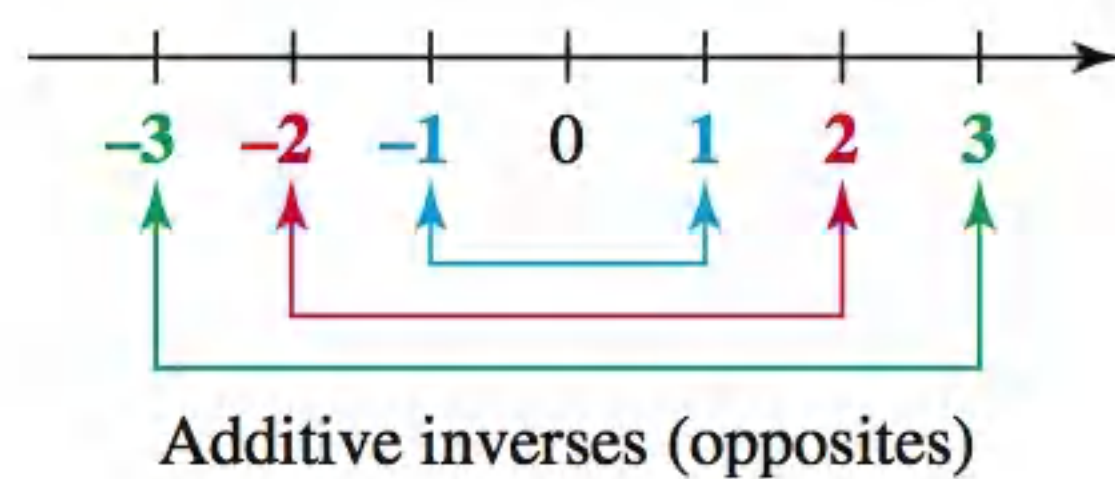
This is true. As shown in **Figure 5** on the previous page, the set of real numbers includes all irrational numbers.

(b) Every rational number is an integer.

This statement is false. Although some rational numbers are integers, other rational numbers, such as  $\frac{2}{3}$  and  $-\frac{1}{4}$ , are not.

Work Problem 6 at the Side. ►

**OBJECTIVE 4** Find additive inverses. Look at the number line in **Figure 6**. For each positive number, there is a negative number on the opposite side of 0 that lies the same distance from 0. These pairs of numbers are *additive inverses*, *opposites*, or *negatives* of each other. For example, 3 and  $-3$  are additive inverses.



**Figure 6**

### Additive Inverse

For any real number  $a$ , the number  $-a$  is the **additive inverse** of  $a$ .

*We change the sign of a number to find its additive inverse. As we shall see later, the sum of a number and its additive inverse is always 0.*

### Uses of the Symbol $-$

The symbol “ $-$ ” can be used to indicate any of the following.

1. A negative number, as in  $-9$ , read “*negative 9*”
2. The additive inverse of a number, as in “ $-4$  is the additive inverse of 4”
3. Subtraction, as in  $12 - 3$ , read “*12 minus 3*”

In the expression  $-(-5)$ , the symbol “ $-$ ” is being used in two ways: the first  $-$  indicates the additive inverse (or opposite) of  $-5$ , and the second indicates a negative number,  $-5$ . Because the additive inverse of  $-5$  is 5,

$$-(-5) = 5.$$

This example suggests the following property.

$$-(-a)$$

For any real number  $a$ ,  $-(-a) = a$ .

6 Decide whether the statement is *true* or *false*. If false, tell why.

- (a) All whole numbers are integers.
- (b) Some integers are whole numbers.
- (c) Every real number is irrational.

### Answers

6. (a) true (b) true  
(c) false; Some real numbers are irrational, but others are rational numbers.